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A Dimensionless Model for Soil Swelling Behaviour

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ABSTRACT: Swelling behaviour of soils is of particular interest for geotechnical structures and the swelling mechanisms have been widely studied. It has been shown that the swelling capacity of a given soil depends on numerous parameters among which its initial dry density, its initial hydration state expressed in terms of saturation degree, water content or suction, the mechanical boundary conditions during the swelling test and the chemical composition of the solution used to hydrate the sample. A comprehensive investigation on the swelling behaviour of a given soil should then include the study of the influence of all these parameters, leading to heavy experimental programs. This paper proposes to tackle the swelling behaviour by means of dimensionless parameters in the manner of fluid mechanics so that the number of experiments required to characterize the swelling behaviour could be significantly reduced. A dimensional analysis is performed and several combinations of the key parameters for the swelling problem are studied in order to obtain different dimensionless parameters. Their validity is assessed using results from the scientific literature. It appears that this innovative approach enables a reasonably good description and prediction of the soil swelling behaviour.

1 Introduction

Swelling behaviour of soils is of particular interest for geotechnical structures and soil structure interactions and it has been extensively studied since the 1950s (e.g. by Holtz & Gibbs, 1956 or by Seed & Chan, 1959). The swelling of soil is governed by physico-chemical aspects of microscale processes related to the adsorption of water molecules onto the clay platelet surfaces. Parcher & Liu (1965) identified several key factors affecting soil swelling including the composition of the soil, the initial water content, water chemistry, the soil structure, the level of compaction and the confining pressure. Attention is usually focused on a reduced number of parameters, typically the confining pressure, the initial hydration state (expressed in terms of water content, saturation degree or suction) and the initial level of compaction (expressed in terms of dry density or void ratio); in order to investigate a simpler problem. Still, understanding the influence of these three parameters requires a significant number of tests, which results are usually processed using statistical methods and empirical relationships in order to allow one to predict the amount of swelling.

In the technical literature, these relationships are often incomplete and do not incorporate all of these three parameters (e.g. in Salas & Serratosa, 1957; Noble, 1966). Even in more recent studies, the amount of swelling is rarely correlated to more than two parameters, which helps to keep the formulation simple and allows it to be represented graphically. For example, Komine (2004) has expressed the swelling strain has a function of initial dry density and vertical stress. Some authors have linked the volume change to more than two parameters using multiple regression methods but this is time consuming and it leads to complex formulations (Yevnin & Zaslavsky, 1970). Moreover, it is difficult to plot relationships in a graphical representation. Even when they incorporated three variables in the swelling equation, Yevnin & Zaslavsky (1970) chose to keep some parameters constant in graphical representations of their solutions.

This paper proposes to characterize soil swelling behaviour by means of dimensional analysis in the manner of fluid mechanics. This approach is commonly used to reduce the number of variables of a problem and consequently the number of experiments needed to characterize a particular problem. The different key parameters of the swelling problem are identified and four combinations corresponding to four possible dimensional analyses are proposed. Then, applying the Pi Buckingham Theorem leads to two new dimensionless numbers, which validity is assessed using data coming from the literature.

2 Dimensional analysis and combination of parameters

Dimensional analysis is a classical approach from the field of fluid mechanics. In fluid mechanics, the governing equations of fluid flow are often quite complex to solve (e.g. Navier stokes equations), and instead, experiments are often used to investigate the flow phenomena. In this context, dimensional analysis is used to reduce the number of independent parameters considered and the number of tests needed to fully characterize a particular problem. The Buckingham Pi theorem states that an equation describing a phenomenon and involving *N* independent parameters and *P* independent dimensions can be reduced to a simpler equation involving *N-P* dimensionless numbers built on the *N* parameters (as explained in Fox & McDonald (1992) for example).

This approach is applied to the soil swelling phenomenon in this paper and as several parameters affect the amount of swelling of a soil, a framework has to be defined. The analysis developed herein aims to provide complementary information on a series of tests performed in oedometric conditions (1D vertical swelling), for constant water chemistry and for a given soil. Under these conditions, the relevant macro scale parameters of the swelling problem are the initial level of compaction (structure in undisturbed soils), expressed either in terms of void ratio or initial dry density; the initial hydration state, expressed either in terms of saturation degree, water content or suction and the level of confinement i.e. vertical stress. Several combinations are then possible to form dimensionless numbers. They are examined in the following.

First of all, saturation degree has not been chosen because expansive soils are known for remaining saturated over wide range of water content or suction making the saturation degree meaningless. It is preferable to use either the initial suction or the initial water content. Note that for the dimensional analysis, it is recommended to avoid parameters which are already dimensionless such as initial void ratio or initial water content. Their definition involving volumes or masses respectively should be considered. Table 1 summarizes the possible combinations and the corresponding number of dimensionless parameters when applying the Pi Buckingham Theorem. Three dimensions are involved: time, length and mass.

Table 1. Various combinations of parameters to describe the swelling problem and corresponding numbers of dimensionless parameters (*N-P*) when applying the Pi Buckingham Theorem. w_o: initial water content, γ_{do}: initial dry density, h_o: initial height of the specimen, Δh: change in height due to swelling, σ_v: vertical stress applied on the specimen, M_{wo}: initial mass of water in the specimen, M_s: mass of solid particles in the specimen, V_{wo}: initial volume of water in the specimen, V_s: volume of solid particles in the specimen, u_{wo}: initial suction, e_o: initial void ratio, *N:* number of independent parameters in the equation of the swelling problem, *P*: number of dimensions, *N-P*: number of dimensionless parameters required to describe the same problem once applied the Pi Buckingham Theorem.

Combination	Initial parameters chosen to describe the swelling problem	Equation of the swelling problem for the dimensional analysis	*N*	*P*	*N-P*
1	*suction, dry density, vertical stress*	$F(u_{wo}, \gamma_{do}, h_o, \Delta h, \sigma_v)=0$	5	3	2
2	*water content, dry density, vertical stress*	$F(M_{wo}, M_s, \gamma_{do}, h_o, \Delta h, \sigma_v)=0$	5	3	2
3	*water content, void ratio, vertical stress*	$F(M_{wo}, M_s, V_{wo}, V_s, h_o, \Delta h, \sigma_v)=0$	6	3	3
4	*suction, void ratio, vertical stress*	$F(u_{wo}, V_{wo}, V_s, h_o, \Delta h, \sigma_v)=0$	6	3	3

Note that *N*, the number of independent parameters, is equal to 5 and 6 for combinations 1 and 3, respectively. Indeed, changing the mass of solid particles of the specimen M_s would inevitably change its dry density γ_{do} (combination 1) and the volume of solid particles V_s (combination 3). The Pi Buckingham theorem states that the same problem can be described using *N-P* dimensionless parameters built on the parameters of the swelling equation so that according to the combination chosen, 2 or 3 dimensionless parameters have to be built. For sake of simplicity, combinations 3 and 4 will be discarded since they lead to 3 dimensionless parameters and attention is focused on combinations 1 and 2.

3 Definition of dimensionless parameters and validation

3.1 Combination 1 (use of initial suction)

Two dimensionless parameters have to be built on the parameters used in the swelling equation. A first known dimensionless number, defined for both combinations 1 and 2 and considered to be relevant herein, is the swelling strain ε_{sw}:

$$\varepsilon_{sw} = \frac{\Delta h}{h_o} \tag{1}$$

The second new dimensionless parameter, called DSP_u (for a Dimensionless Swelling Parameter involving suction), is defined using u_{wo}, γ_{do}, h_o and σ_v as follows:

$$DSP_u = (\gamma_{do}.h_o)^a (\sigma_v)^b (u_{wo})^c \quad (2)$$

with $a + b + c = 0$ for DSP_u to be dimensionless. From a mathematical point of view, a, b and c can be any real but for the sake of simplicity, only integer values are considered and, even if it reduces the number of degree of freedom of the model, a is taken equal to 1. Note that in DSP_u, h_o is not included as an independent parameter but it is included to turn γ_{do} into a pressure. According to the Buckingham Pi theorem, the swelling problem as formulated in Table (1) can now be written as:

$$\varepsilon_{sw} = f(DSP_u) \quad (3)$$

The only manner to assess the validity of equation (3) is to use experimental data and to plot the results in terms of swelling strain versus DSP_u. If a correlation can be found between these two entities with a reasonable scattering, it could be concluded that the dimensional approach can be applied to the soil swelling behaviour. A data set coming from Villar (2000) who has performed several series of swelling tests on a compacted bentonite has been used. Her data include all the parameters required to calculate DSP_u so that the evolution of the swelling strain versus DSP_u could be plotted (Figure (1)). The adjustable factors b and c have been calibrated (a=1, b=-2 and c=1) to obtain the best correlation factor R^2. The overall idea when defining DSP_u was that a higher DSP_u, should correspond to a greater amount of swelling. This is why a and c are chosen in the positive range whereas b is negative.

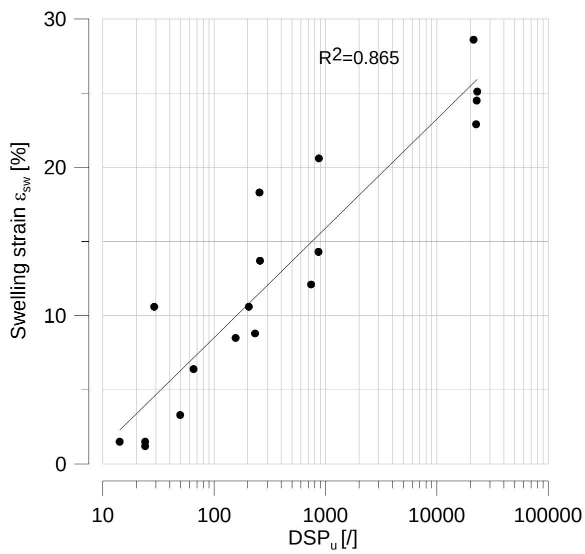

Figure 1. Evolution of swelling strain ε_{sw} vs. dimensionless parameter DSP_u for data after Villar (2000). DSP_u defined with a=1, b=-2 and c=1. 18 test results.

A relationship is clearly visible and a logarithmic trend (linear in semi logarithmic scale) is used to fit the data. Any curve fitting could be used provided that it gives a good correlation factor. Note that the logarithmic relationship is convenient for its simplicity. The scatter is reasonably low (standard deviation from the trend line of 2.6) and a relatively high correlation factor is found (R^2 = 0.865) so that it can be concluded that the dimensionless number DSP_u can be used in a satisfactory way to describe the swelling problem; at least for this particular set of data, for which Equation (3) can be rewritten as follows:

$$\varepsilon_{sw} = 3.2 \cdot \ln(DSP_u) - 6.2 \quad (4)$$

As well known, the amount of swelling depends on the soil mineralogy so that some soil parameter should appear in the dimensionless equation of the swelling behaviour. It not known yet if the factor b in the definition of the dimensionless number is a soil parameter, this is still to be investigated. However, the coefficients of the logarithmic relationship (e.g. 3.198 and -6.199 in Equation (4)) certainly are. Indeed, they directly describe the response of the soil upon wetting.

The significant outcomes are that only two independent parameters are required to describe the swelling behaviour and combining the key parameters of the problem in only one parameter DSP_u leads to a significant reduction of the number of tests.

3.2 Combination 2 (use of initial water content)

The early studies published in the literature are often the most comprehensive, and more commonly, the water content is used to describe the initial hydration state. In the last three decades, the technology for suction measurement and control has improved so that studies based on suction data are now more common but the experimental programs are usually less comprehensive and focused on other aspects of the behaviour (e.g. cyclic wetting/drying). As previously, only 2 dimensionless parameters are required to describe the swelling issue once applied the Pi Buckingham Theorem, one of which being the swelling strain ε_{sw}. The second dimensionless parameter to be defined for this specific combination, using M_{wo}, M_s, γ_{do}, h_o and σ_v is called DSP_w (for the Dimensionless Swelling Parameter involving water content). It is written as:

$$DSP_w = \left(\frac{\gamma_{do} \cdot h_o}{\sigma_v}\right)^a \left(\frac{M_s}{M_{wo}}\right)^b = \left(\frac{\gamma_{do} \cdot h_o}{\sigma_v}\right)^a \left(\frac{1}{w_o}\right)^b \quad (5)$$

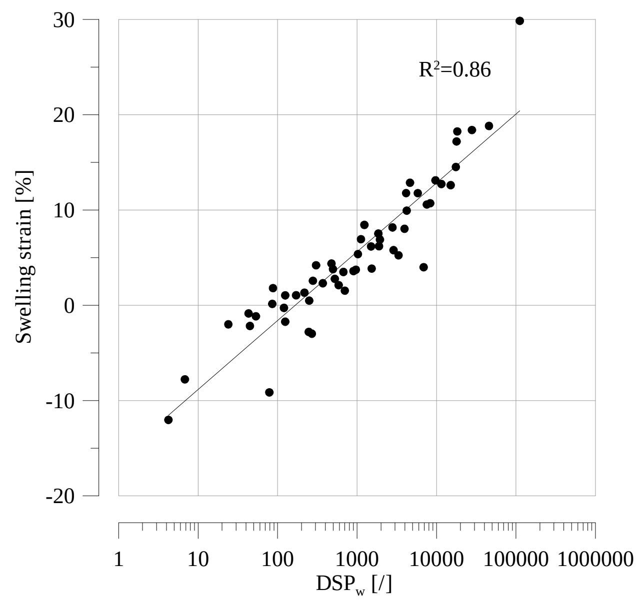

Figure 2. Evolution of swelling strain ε_{sw} vs. dimensionless parameter DSP_w for data after Yevnin & Zaslavsky (1970). 56 test results.

From a dimensional point of view, ***a*** and ***b*** are real numbers and can have any value since the ratios M_s/M_{wo} and $(\gamma_{d0}.h_o)/\sigma_v$ are already dimensionless. As previously, for the sake of simplicity and even if it reduces the degrees of freedom of the problem, ***a*** is taken equal to 1 and positive integer values are only considered for ***b***. As previously, the number of parameters would be significantly reduced if the swelling equation (3) can be formulated as a relationship involving two independent entities ε_{sw} and DSP_w:

$$\varepsilon_{sw} = f(DSP_w) \quad (6)$$

The validity of equation (6) is assessed using data after Yevnin & Zaslavsky (1970) who have performed a large series of swelling tests. The initial water content, initial dry density and vertical stress are provided for each test so that it is possible to calculate DSP_w. As before, a good correlation (semi logarithmic trend) can be found between ε_{sw} and DSP_w (Figure (2)) validating the dimensional approach for swelling tests. For this set of data, a correlation of 0.86 and a standard deviation from the trend line of 1.85 are found for ***a***=1 and ***b***=5. The best fitting, once again logarithmic, is:

$$\varepsilon_{sw} = 3.1 \cdot \ln(DSP_w) - 16.1 \quad (7)$$

Once again, it is possible to use the dimensional approach to predict the swelling behaviour in a simpler way.

Regarding combination 2, the use of initial water content by itself can be discussed. Indeed, from a phenomenological point of view, the initial water content is maybe not enough and the ratio of the final water content over the initial one would make more sense. However, this raises the issue of the dependency of DSP_w

on ε_{sw}. Indeed, the dimensionless parameter (now noted DSP_{wf} to avoid any confusion with DSP_w) would become:

$$DSP_{wf} = \left(\frac{\gamma_{do} \cdot h_o}{\sigma_v}\right)^a \left(\frac{w_f}{w_o}\right)^b \quad (8)$$

With w_f the final water content and b positive. It can easily be shown that w_f depends on ε_{sw} and the swelling equation becomes:

$$\varepsilon_{sw} = f(DSP_{wf}(\varepsilon_{sw})) \quad (9)$$

where the dependence issue appears clearly.

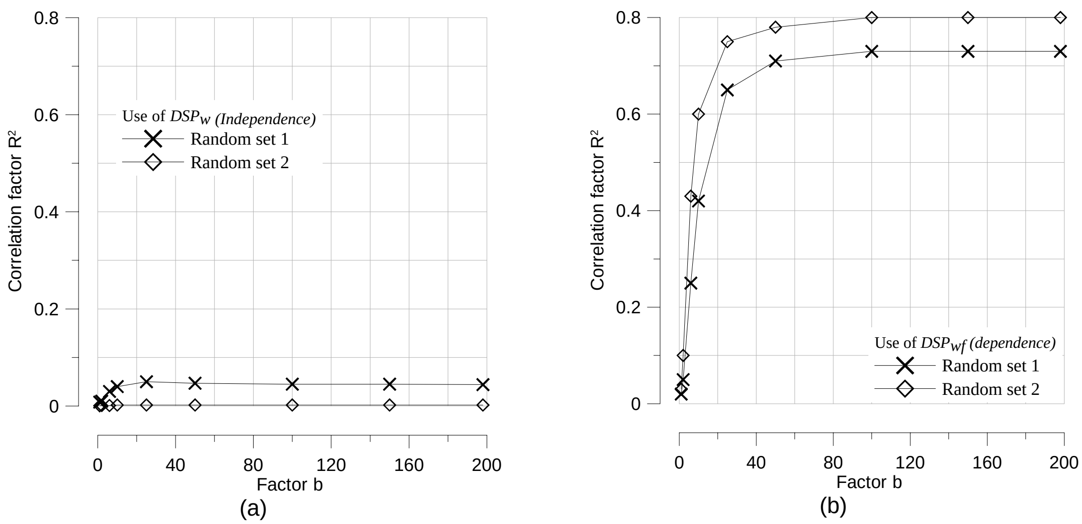

Figure 3. Evolution of the correlation factor R^2 with respect to factor *b* used in DSP_w (a) and in DSP_{wf} (b). Unlike DSP_w, DSP_{wf} is dependent on ε_{sw}.

The consequence of such dependence has been studied using two sets of swelling strains randomly generated. The two sets (called set1 and set2) have an average swelling strain of 26.6 and 60.3 and a standard deviation of 8.2 and 26.3, respectively. These random values of swelling strain have been associated to the tests performed by Yevnin and Zaslavsky (1970) even though there is absolutely no physical correlation between the test parameters (w_o, γ_{do}, h_o, σ_v) and the result of the test in terms of swelling strain. Then, the dimensional approach has been applied to the random experimental data using both DSP_w and DSP_{wf}. As previously, the factor *b* in equations (8) and (5) has been adjusted to get the best correlation factor. Figure (3a) and (3b) show the evolution of the correlation factor R^2 with respect to factor *b* used in DSP_w and in DSP_{wf}, respectively.

It appears clearly in Figure (3a) that, as expected, no physical correlation can be found between the random swelling strain and DSP_w since this latter is defined independently of the results ($R^2<0.05$). On the contrary, when the dimensionless parameter DSP_{wf} is dependent on the results, the correlation factor can reach values up to 0.80 which is enough to consider that a physical correlation may exist. It is then of prime importance to build a dimensionless parameter independently of the result values to produce a reliable model.

4 Use of the dimensionless model

Some experimental data coming from the literature have been used to assess whether a dimensional analysis, leading to a dimensionless model, can be used to link the amount of swelling to three key parameters in a simple way. It has been shown that it can work but this is not an absolute proof. Consequently, it is necessary to check, before applying it in any situation, that the experimental results obtained can be properly described using the proposed dimensionless number. If the correlation obtained from an appropriate set of measured data is satisfactory, further predictions can reasonably be made. From a statistical point of view, 15 tests can be considered as a reasonable set of calibration data.

For example, the data set from Yevnin & Zaslavsky (1970) of 56 data have been divided in 2 sub sets of 28 data, one used for the calibration of the model and the second one used to check the quality of the prediction. The dimensionless parameter DSP_w has been used with *a*=1 and *b*=5. The model obtained (Figure 4a), with a correlation factor R^2 of 0.89, is:

$$\varepsilon_{sw} = 3.2 \cdot \ln(DSP_w) - 16.6 \quad (10)$$

The comparison between predicted and measured swelling strains can be seen in Figure 4(b). The prediction appears to be relatively good.

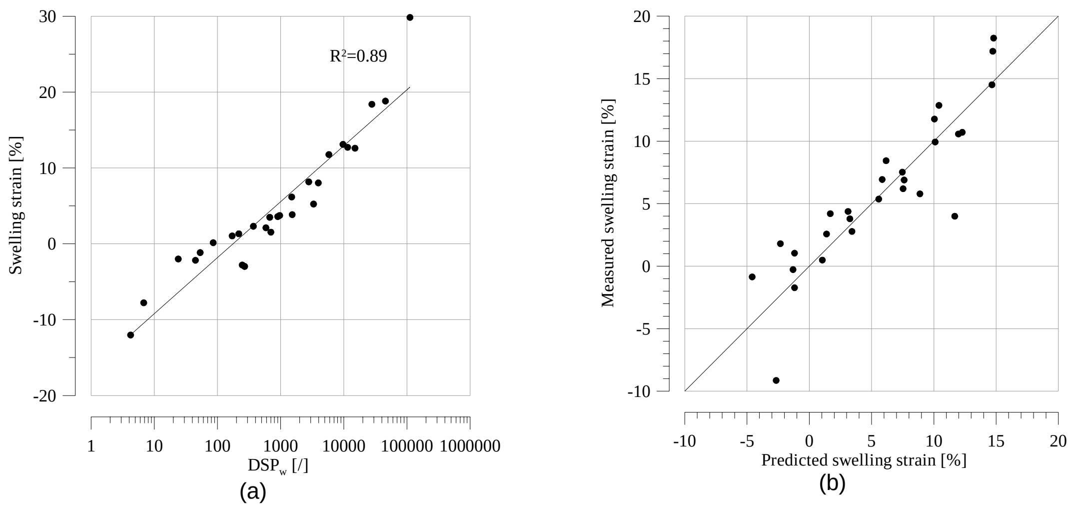

Figure 4. (a) Calibration of the dimensionless model: evolution of the swelling strain versus DSP_w for data after Yevnin & Zaslavsky (1970). Model is defined by ε_{sw}*=3.2.ln(*DPS_w*)-16.6* with *b=5*.(b) Use of the model for prediction: Measured swelling strain versus predicted swelling strain.

5 Conclusions

The swelling capacity of a soil is known to depend on several key variables. More specifically, when the behaviour of a given soil is investigated, for example in commonly adopted oedometric conditions with a given soil-water chemistry, the amount of swelling mainly depends on the initial hydration state, on the initial compaction level (or structure for undisturbed soils) and on the confining pressure. Correlating the volume change to all these three parameters is seldom done and usually leads to complex formulations and a lack of graphical representation. A dimensional analysis undertaken in the manner of fluid mechanics is presented in this paper and the initial hydration state, the initial level of compaction and the vertical stress are combined into dimensionless numbers. Two data sets from the literature have been used to assess the validity of the dimensionless description of soil swelling behaviour and its possible use as a predictive model. It is pointed out that the independency between swelling strain and dimensionless parameter is vital to obtain a reliable model. Reasonably good correlations have been found between the swelling strain and the dimensionless parameters validating the application of dimensional analysis to soil swelling. This innovative approach helps significantly to reduce the number of tests to be performed in the laboratory to fully capture the swelling behaviour of a soil. Moreover, combining three key parameters into one unique parameter leads to an easier graphical representation of the results and implementation of the model in a numerical code. More work has to be done to understand completely if the factors a, b and c used in the definition of the dimensionless parameters are soil dependent.

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